9/1/2022

Hillstone Primary School

Calculation Guidance Policy 2022

Calculation Guidance – for HPS taken from NCETM guidance

Develop children’s fluency with basic number facts

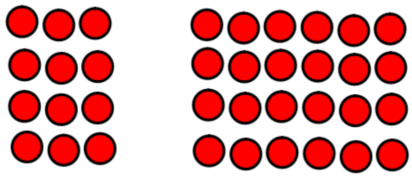
Fluent computational skills are dependent on accurate and rapid recall of basic number bonds to 20 and times-tables facts. Spending a short time every day on these basic facts quickly leads to improved fluency.

This is **not** meaningless rote learning; rather, this is an important step to developing conceptual understanding through identifying patterns and relationships between the tables (for example, that the products in the 6× table are double the products in the 3× table).

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

3 x 1 = 3	6 x 1 = 6
3 x 2 = 6	6 x 2 = 12
3 x 3 = 9	6 x 3 = 18
3 x 4 = 12	6 x 4 = 24
3 x 5 = 15	6 x 5 = 30
3 x 6 = 18	6 x 6 = 36
3 x 7 = 21	6 x 7 = 42
3 x 8 = 24	6 x 8 = 48
3 x 9 = 27	6 x 9 = 54
3 x 10 = 30	6 x 10 = 60

Array



This has helped children develop a strong sense of number relationships, an important prerequisite for procedural fluency.

Children who learn their multiplication tables in this order have opportunities to make connections:

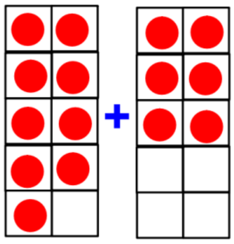
×10	×5	×2	×4	×8	×3	×6	×9	×7

Develop children’s fluency in mental calculation

Efficiency in calculation requires **having a variety of mental strategies**. In particular teachers should emphasise **the importance of 10** and partitioning numbers to bridge through 10.

For example:

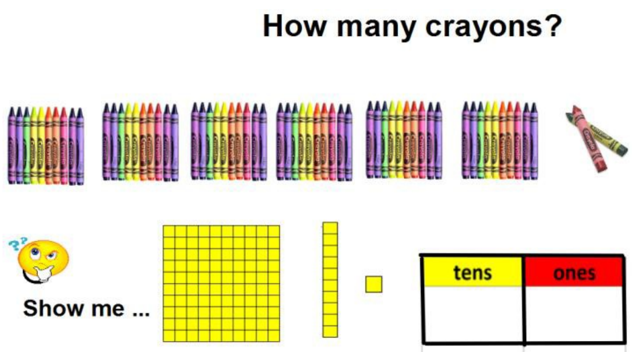
9 + 6 = 9 + 1 + 5 = 10 + 5 = 15.



Shanghai teachers refer to “**magic 10**”. It is helpful to make a 10 as this makes the calculation easier.

Develop fluency in the use of formal written methods

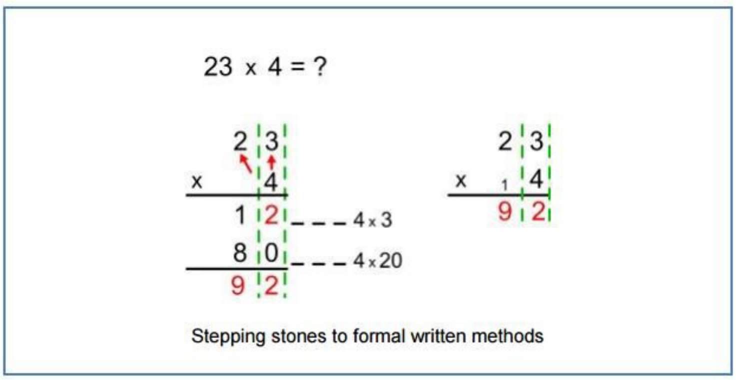
Teaching column methods for calculation provides the opportunity to develop both procedural and conceptual fluency. However, teachers must ensure children understand the structure of the mathematics presented in the algorithms, with a **particular emphasis on place value**. Children in Y1 and Y2 explore grouping objects in order to count them and come to the conclusion that grouping in tens is easy to count. They make base ten from resources such as straws, then Unifix cubes, prior to being introduced to structured base ten equipment.



Children use base ten apparatus in class, see it illustrated in textbooks/workbooks and draw it in their maths books to support the development of fluency and understanding.

Informal methods of recording calculations are an important stage to help children develop fluency with formal methods of recording. These are used for a short period, as an interim step to help children understand the internal logic of formal methods of recording calculations. They are stepping stones to formal written methods.
However mental methods should always be used when more efficient.

Here is an example from a Shanghai textbook:



Calculation Guidance – for HPS taken

Develop children’s understanding of the = symbol

The symbol **=** is an assertion of equivalence. If we write:

$$\mathbf{3 + 4 = 6 + 1}$$

Then we are saying that what is on the left of the **=** symbol is necessarily equivalent to what is on the right of the symbol. But many children interpret = as being simply an instruction to evaluate a calculation, as a result of always seeing it used thus:

$$\mathbf{3 + 4 = \quad 5 \times 7 = \quad 16 - 9 =}$$

If children only think of = as meaning “work out the answer to this calculation” then they are likely to get confused by empty box questions such as:

$$\mathbf{3 + \square = 8}$$

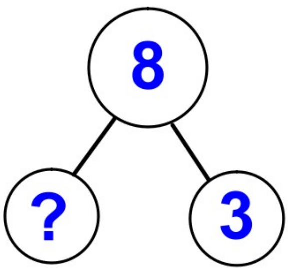
Later they are very likely to struggle with even simple algebraic equations, such as:

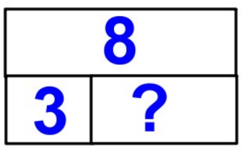
$$\mathbf{3y = 18}$$

One way to model equivalence such as:

$$\mathbf{2 + 3 = 5}$$

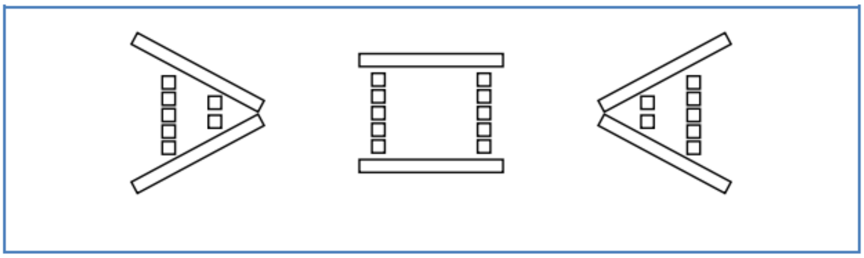
is to use balance scales (see illustrations below). Teachers should vary the position of the = symbol and include empty box problems from Year 1 to deepen children’s understanding of the = symbol.





Teach inequality alongside teaching equality

To help young children develop their understanding of equality, they also need to develop understanding of inequality. From Y2 inequality should be taught before, or at the same time as, equality (as in Shanghai and Singapore). One way to introduce the < and > signs is to use rods and cubes to make a concrete and visual representations such as:



to show that 5 is greater than 2 (5 > 2), 5 is equal to 5 (5 = 5), and 2 is less than 5 (2 < 5). Balance scales can also be used to represent inequality.

Incorporating both equality and inequality into examples and exercises can help children develop their conceptual understanding.

For example, in this empty box problem children have to decide whether the missing symbol is :

$$5 + 7 \square 5 + 6$$

An activity like this encourages children to develop their mathematical reasoning:

"I know that 7 is greater than 6, so 5 plus 7 must be greater than 5 plus 6" and shows depth of understanding.

Asking children to decide if number sentences are **true or false** also helps develop mathematical reasoning.

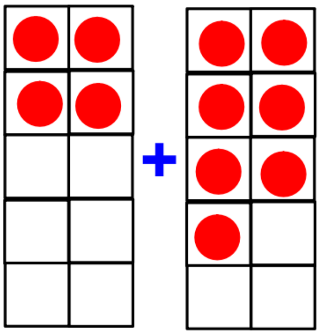
For example, in discussing this statement:

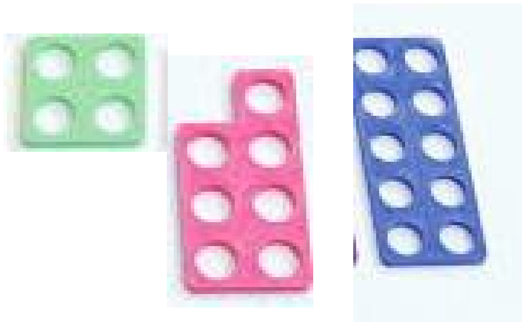
$$4 + 6 + 8 > 3 + 7 + 9$$

a child might reason that "4 plus 6 and 3 plus 7 are both 10. But 8 is less than 9. Therefore 4 + 6 + 8 must be less than 3 + 7 + 9, not more than 3 + 7 + 9". In both these examples the numbers have been deliberately chosen to allow the children to establish the answer without actually needing to do the computation. This emphasises further the importance of mathematical reasoning and the importance of careful selection of numbers chosen by teachers when setting tasks.

Don ’t count, calculate

Young children benefit from being helped at an early stage (Y1) to start calculating, rather than relying on ‘counting on’ as a way of calculating. For example, with a sum such as: 4 + 7 = Rather than starting at 4 and counting on 7, children could use their knowledge and bridge to 10 to deduce that because 4 + 6 = 10, so 4 + 7 must equal 11.





Look for pattern and make connections

As in Shanghai and Singapore, teachers should use concrete resources (models) and visual representations (images) of the mathematics. Understanding, however, does not happen automatically, **children need to reason by and with themselves and make their own connections** (not be shown or told by the teacher).

Children should get into good habits early (at least from Year 1) in terms of reasoning and looking for patterns and connections in the mathematics. The question “**What’s the same, what’s different**?” should be used frequently to make comparisons. For example “What’s the same, what’s different between the three times table and the six times table?”

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

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3 x 6 = 18	6 x 6 = 36
3 x 7 = 21	6 x 7 = 42
3 x 8 = 24	6 x 8 = 48
3 x 9 = 27	6 x 9 = 54
3 x 10 = 30	6 x 10 = 60

Use intelligent practice

Children should engage in a significant amount of practice of mathematics through class- and homework exercises. However, in designing [these] exercises, the teacher is advised to **avoid**

mechanical repetition and to create an appropriate path for practising the thinking process with **increasing creativity** (Gu, 1991). The practice that children engage in should provide the opportunity to develop both **procedural and conceptual fluency**. Children should be required **to reason and make connections between calculations**. The connections made improve their fluency.

2 × 3 =	6 × 7 =	9 × 8 =
2 × 30 =	6 × 70 =	9 × 80 =
2 × 300 =	6 × 700 =	9 × 800 =
20 × 3 =	60 × 7 =	90 × 8 =
200 × 3 =	600 × 7 =	900 × 8 =

Shanghai Textbook Grade 2 (aged 7/8)

<u>Use empty box problems</u>

Empty box problems are a powerful way to help children develop a strong sense of number through intelligent practice. They provide the opportunity for reasoning and finding easy ways to calculate.

They enable children to practise procedures, whilst at the same time thinking about conceptual connections.

A sequence of examples such as:

3 + □ = 8

3 + □ = 9

3 + □ = 10

3 + □ = 11

helps children develop their understanding that the = symbol is an assertion of equivalence, and invites children to spot the pattern and use this to work out the answers.

This sequence of examples does the same at a **deeper level**:

3 × □ + 2 = 20

3 × □ + 2 = 23

3 × □ + 2 = 26

3 × □ + 2 = 29

3 × □ + 2 = 35

Children should also be given examples where the empty box represents the operation, for example:

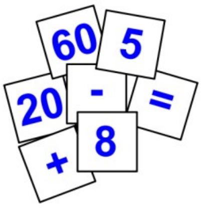
4 × 5 = 10 □ 10

6 □ 5 = 15 + 15

6 □ 5 = 20 □ 10

8 □ 5 = 20 □ 20

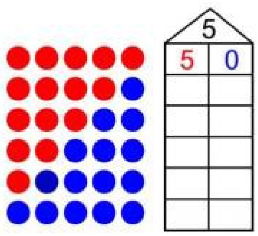
8 □ 5 = 60 □ 20



These examples also illustrate the careful use of variation to help children develop **both procedural and conceptual fluency.**

<u>Expose mathematical structure and work systematically</u>

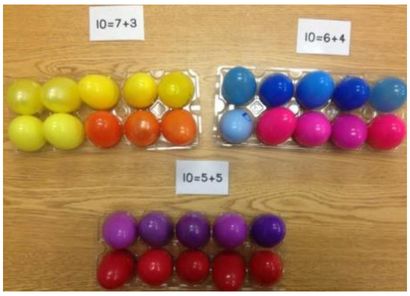
Developing **instant recall alongside conceptual understanding of number bonds to 10** is vital. This begins by learning number bonds to five. And is supported through the use of models/images such as the examples illustrated below:

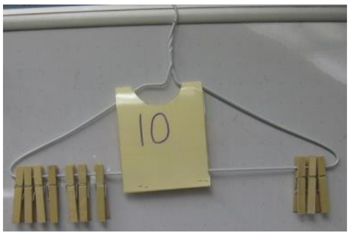


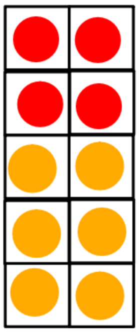
Shanghai Textbook Grade 1 (aged 6/7)

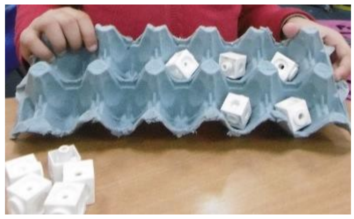
The image lends itself to seeing pattern and working systematically and children can connect one number fact to another and be certain when they have found all the bonds to 5.

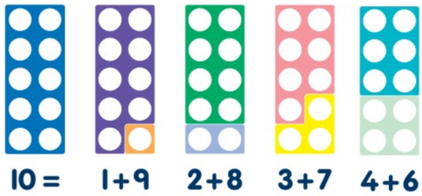
Using other structured models such as tens frames, part whole models or bar models can help children to reason about mathematical relationships.

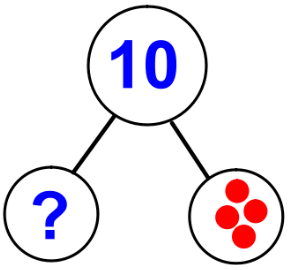


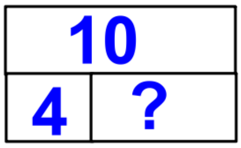


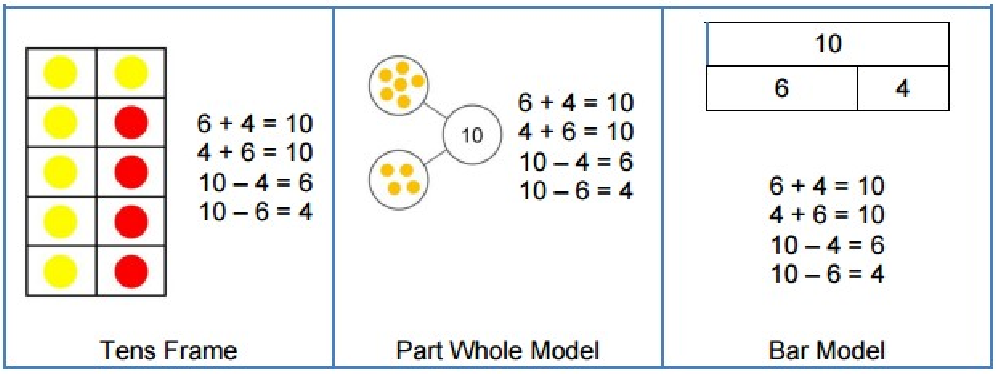


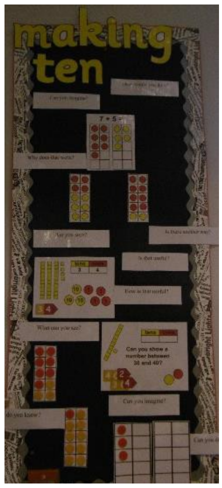












<u>Tens Frame Part Whole Model Bar Model</u>

Connections between these models should be made, so that children understand the same mathematics is represented in different ways. Asking the question **"What's the same what's different?"** has the potential for children to draw out the connections. Illustrating that the same structure can be applied to any numbers helps children to generalise mathematical ideas and build from the simple to more complex numbers, recognising that the structure stays the same; it is only the numbers that change.

10	247	6.2
6 | 4	173 | 74	3.4 | 2.8
6 + 4 = 10	173 + 174 = 247	3.4 + 2.8 = 6.2
4 + 6 = 10	74 + 173 = 247	2.8 + 3.4 = 6.2
10 – 6 = 4	247 – 173 = 74	6.2 – 3.4 = 2.8
10 – 4 = 6	247 – 74 = 173	6.2 – 2.8 = 3.4

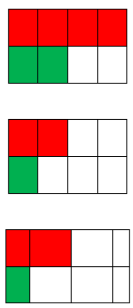
For example:

Move between the concrete and the abstract (CPA Approach)

Children's conceptual understanding and fluency is strengthened if they experience **concrete, visual and abstract (Concrete, pictorial, abstract or models, images & symbolic) representations of a concept during a lesson**. Moving between the concrete and the abstract helps children to connect abstract symbols with familiar contexts, thus providing the opportunity to **make sense of, and develop fluency** in the use of, abstract symbols.

For example, in a lesson about addition of fractions children could be asked to draw a picture to represent the sum: $\frac{1}{4} + \frac{1}{8} = \frac{3}{8}$.

Alternatively, or in a subsequent lesson, they could be asked to discuss which of three visual images correctly represents the sum, and to explain their reasoning:



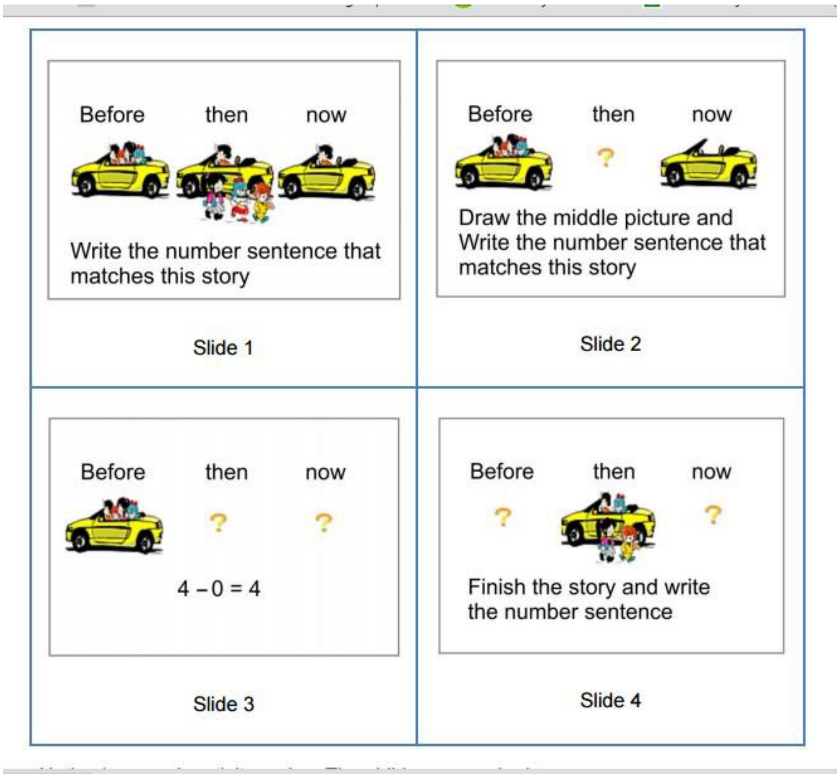
Contextualise the mathematics

Lessons begin with a problem to solve/an anchor task, which is linked to a context. For example a lesson about addition and subtraction could start with this contextual story: “There are 11 people on a bus. At the next stop 4 people get on. At the next stop 6 people get off. How many are now on the bus?” (Children should have the opportunity to explore such problems with objects and pictures.)

This helps children develop their understanding of the concepts of addition and subtraction. But during the lesson the teacher should keep returning to the story.

For example, if the children are thinking about this calculation 14 – 8 then the teacher should ask the children: “What does the 14 mean? What does the 8 mean?”, expecting that children will answer: “There were 14 people on the bus, and 8 is the number who got off.” Then asking the children to interpret the meaning of the terms in a sum such as 7 + 7 = 14 will give a good assessment of the **depth of their conceptual understanding** and their ability to link the concrete and abstract representations of mathematics.

The four slides below are taken from a lesson delivered by one of the Shanghai teachers (Li Dong) who visited England during the Shanghai exchange:



Notice how each activity varies.

The children are asked to:

Slide 1: Start with the story (concrete) and write the number sentence (abstract).

Slide 2: Start with the story (concrete) and complete it. Then write the number sentence (abstract).
Slide 3: Start with the number sentence (abstract) and complete the story (concrete).

Slide 4: Start with part of the story, complete two elements of it (concrete with challenge) and then write the number sentence (abstract).

The children move between the concrete and the abstract and back to the concrete, with an increasing level of difficulty.

Use questioning to develop mathematical reasoning

Teachers' questions in mathematics lessons are often asked in order to find out whether children can give the right answer to a calculation or a problem. But in order to develop children's conceptual understanding and fluency there needs to be **a strong and consistent focus on questioning that encourages and develops their mathematical reasoning.** This can be done simply by asking children to explain how they worked out a calculation or solved a problem, and to compare and contrast different methods that are described.

Children quickly come to expect that they need to explain and justify their mathematical reasoning (as found in reception and Y1), and they soon start to do so automatically – and enthusiastically.

Some calculation strategies are more efficient and the teacher should **scaffold children's thinking to guide them to the most efficient methods**, whilst at the same time valuing their own ideas.

Rich questioning strategies include:

"What's the same, what's different?" In this sequence of expressions, what stays the same each time and what's different?

23 + 10	23 + 20	23 + 30	23 + 40

Discussion of the variation in these examples can help children to identify the relationship between the calculations and hence to use the pattern to calculate the answers.

"Odd one out" Which is the odd one out in this list of numbers: 24, 15, 16 and 22?

This encourages children to apply their existing conceptual understanding.

Possible answers could be:

"15 is the odd one out because it's the only odd number in the list."

"16 is the odd one out because it's the only square number in the list."

"22 is the odd one out because it's the only number in the list with exactly four factors."

If children are asked to identify an 'odd one out' in this list of products:

24 × 3 **36 × 4** **13 × 5** **32 × 2**

they might suggest:

"36 × 4 is the only product whose answer is greater than 100."

"13 × 5 is the only product whose answer is an odd number."

"Here's the answer. What could the question have been?" Children are asked to suggest possible questions that have a given answer.

For example, in a lesson about addition of fractions, children could be asked to suggest possible ways to complete this sum:

$$\square + \square = \frac{3}{4}$$

Identify the correct question

Here children are required to select the correct question:

A 3.5m plank of wood weighs 4.2 kg

The calculation was:

3.5 ÷ 4.2

Was the question:

a. How heavy is 1m of wood?

b. How long is 1kg of wood?

True or False

Children are given a series of equations are asked whether they are true or false:

4 × 6 = 23

4 × 6 = 6 × 4

12 ÷ 2 = 24 ÷ 4

12 × 2 = 24 × 4

Children are expected to reason about the relationships within the calculations rather than calculate.

Greater than, less than or equal to >

$$3.4 \times 1.2 \bigcirc 3.4 \qquad 5.76 \bigcirc 5.76 \div 0.4 \qquad 4.69 \times 0.1 \bigcirc 4.69 \div 10$$

These types of questions are further examples of intelligent practice where conceptual understanding is developed alongside the development of procedural fluency. They also give pupils who are, to use **Ofsted's phrase, rapid graspers the opportunity to apply their understanding in more complex ways**.

<u>Expect children to use correct mathematical terminology and to express their reasoning in complete sentences</u>

The quality of children's mathematical reasoning and conceptual understanding is significantly enhanced if they are consistently expected to use **correct mathematical terminology** (e.g. saying 'digit' rather than 'number') and to explain their mathematical thinking in complete sentences. Core vocabulary for each year group can be found on medium term planning and key terminology for the week should be displayed in classrooms.

<u>I say, you say, you say, you say, we all say</u>

This technique enables the teacher to provide a sentence stem for children to communicate their ideas with **mathematical precision and clarity.** These sentence structures often express key conceptual ideas or generalities and provide a framework to embed conceptual knowledge and build understanding.

For example:

If the rectangle is the whole, the shaded part is one third of the whole.

Having modelled the sentence, the teacher then asks individual children to repeat this, before asking the whole class to chorus chant the sentence. This provides children with a valuable sentence for talking about fractions. Repeated use helps to embed key conceptual knowledge.

Another example is where children fill in the missing parts of a sentence; varying the parts but keeping the sentence stem the same.

For example:

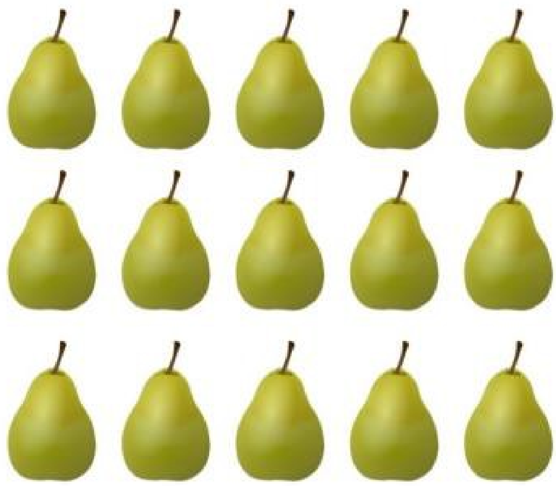
There are 12 **stars**. $\frac{1}{3}$ of the **stars** is equal to **4 stars**

Children use the same sentence stem to express other relationships. For example:

There are 12 **stars**. $\frac{1}{4}$ of the **stars** is equal to **3 stars**

There are 12 **stars**. $\frac{1}{2}$ of the **stars** is equal to **6 stars**

Similarly:



There are **15 pears**. $\frac{1}{3}$ of the **pears** is equal to **5 pears**

There are **15 pears**. $\frac{1}{5}$ of the **pears** is equal to **3 pears**

When talking about fractions it is important to make reference to the whole and the part of the whole in the same sentence. The above examples help children to get into the habit of doing so.

Another example is where a mathematical generalisation or “rule” emerges within a lesson.

For example:

“When adding 10 to a number, the ones digit stays the same.”

This is repeated in chorus using the same sentence, which helps to embed the concept.

<u>Identify difficult points/common misconceptions</u>

Common misconceptions need to be identified and anticipated when lessons are being planned and these **need to be an explicit part of the teaching,** rather than the teacher just responding to children’s difficulties if they happen to arise in the lesson.

The teacher should be **actively seeking to uncover possible difficulties** because if one child has a difficulty it is likely that others will have a similar difficulty. Difficult points also give an opportunity to reinforce that we learn most by working on and through ideas with which we are not fully secure
or confident. Discussion about difficult points can be stimulated by asking children to share thoughts about their own examples when these show errors arising from insufficient understanding.

For example:

$$\frac{2}{14} - \frac{1}{7} = \frac{1}{7}$$

Date reviewed: September 2022
Date of next review: September 2023